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NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL WEATHER SERVICE
NATIONAL METEOROLOGICAL CENTER

OFFICE NOTE 357

CAUSE AND CURE OF THE SPURIOUS NORTHWARD BIAS
INDUCED BY HURRICANE MODEL SPINUP TECHNIQUES

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This is an unreviewed manuscript, primarily
intended for informal exchange of information
among NMC staff members.

1.0 Background

In recent years, one of the techniques used by hurricane forecasting models is to initialize with an axisymmetric vortex. The large scale flow is taken from a conventional analysis or forecast and a symmetric idealized vortex is superimposed upon this flow. Once such a well defined vortex is implanted, the fine mesh, primitive equation model can then forecast the motion of the storm. This paper will attempt to analyse some of the impact such a spinup has upon the model prediction. The questions addressed are basically ones concerning the initialization technique, not the atmosphere or even the model behavior. The arguments are admittedly linear estimates to highly complex processes. The implicit assumption is that, at the very least, the linear effects must be modeled correctly in order to achieve proper performance.

It now appears that a symmetric vortex leads to an erroneous northward motion during the first 12 hours of the model forecast. This has been the experience at NMC using the Movable Fine Mesh (MFM, Hovermale and Livezey, 1977) and the Quasi-Lagrangian Model (QLM, Mathur, 1983). The MFM changed its initialization technique to deal with this problem, using an asymmetric vortex, formed by running the model for 24 hours (forecast time), in a quiet atmosphere so as to develop the asymmetries. Since generating such a spinup required considerable computer resources a library of these spinups were generated, one for each five degrees of latitude. The MFM then interpolated to the desired latitude. This meant that the spinup storm did not resemble the real storm, indeed, all MFM spinups differed only by the beta effect due to latitude.

The QLM, however, returned to the symmetric storm formulation in order to match the spinup with the actual storm size and structure, and to avoid interpolating to the latitude. The QLM's initial 12h forecast error, however, exhibits a substantial northward bias, leading to questions about the wisdom of this choice. It appears now that using the symmetric spinup was responsible for much of the QLM's average error during 1988; not only for the 12h forecasts but for all time periods. The technique presented here attempts to retain the benefits of linking the model storm with the real storm, while avoiding the problems in the MFM method.

An example of the problem being investigated is shown in Figure 1. Figure 1 is a typical example showing an operational QLM track forecast for hurricane Gilbert on September 10, at 12z, 1988. The model initially forecasts the storm to move northwest for about 12 hours, then the forecast track becomes more westerly and the forecast parallels the actual track of the storm motion.

For some other storms, the QLM's initial 12h forecast error occasionally caused the model to move into a different flow regime, producing very large errors at longer time periods.

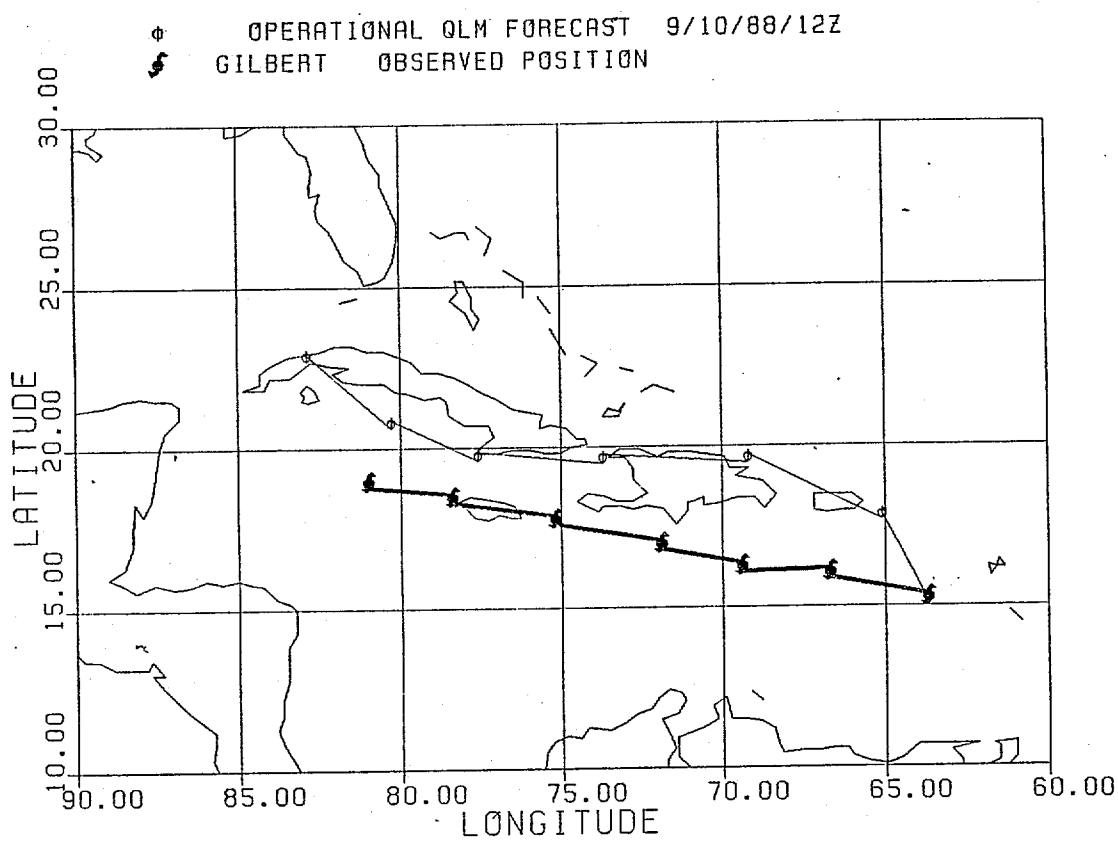


Figure 1. Gilbert 88/9/10/12z. QLM forecast with symmetric spinup.

2.0 Description of the effect

To describe what is happening, we will assume that a circular symmetric vortex is added to a flow field where there is currently no indication of such a circulation. We will eliminate from consideration all those variables except the one suspected of causing this northward bias. Under such idealized conditions, it may be possible to determine the cause and extent of this phenomena.

Assume first that the storm motion can be described by the divergent barotropic vorticity equation.

$$\frac{\partial \zeta}{\partial t} = -\vec{V} \cdot \nabla (\zeta + f) - \omega \frac{\partial \zeta}{\partial p} - (\zeta + f) \nabla \cdot \vec{V} + \vec{k} \cdot \left(\frac{\partial \vec{V}}{\partial p} \times \nabla \omega \right) + \vec{k} \cdot (\nabla \times \vec{F}) \quad (1)$$

The storm will go in that direction where the change in vorticity is a maximum.

Now, assume we are dealing with a quiet, non-divergent, frictionless atmosphere. Onto this quiet atmosphere, we superimpose a symmetric vortex described by:

$$V_s = C / \sqrt{r} \quad (2)$$

where V_s is the tangential wind, r is the distance from the center, and C is a constant. We also ignore the tilting term and the azimuthally symmetric terms which, as Holland (1983) states "may cause an expansion or contraction of the vortex rings surrounding the cyclone but cannot contribute to their net translation". These assumptions basically reduce the motion to that due to the variation of the coriolis force (the beta effect). Then the direction of motion of the storm reduces to:

$$\frac{\partial \zeta}{\partial t} = -\vec{V} \cdot \nabla (\zeta + f) = -V_n \beta = -\beta V_s (\sin \theta) \quad (3)$$

where θ is the angle of motion, measured counterclockwise from north, and V_n denotes the northward wind. Since the maximum for equation (3) occurs at $\theta=90^\circ$, the storm moves straight west. The speed of the storm is obtained by dividing by the rate of change of vorticity in the direction of motion, or:

$$speed = - \frac{\partial \zeta / \partial t}{\partial \zeta / \partial r} \quad (4)$$

From equation (2), and the definition of vorticity, we know,

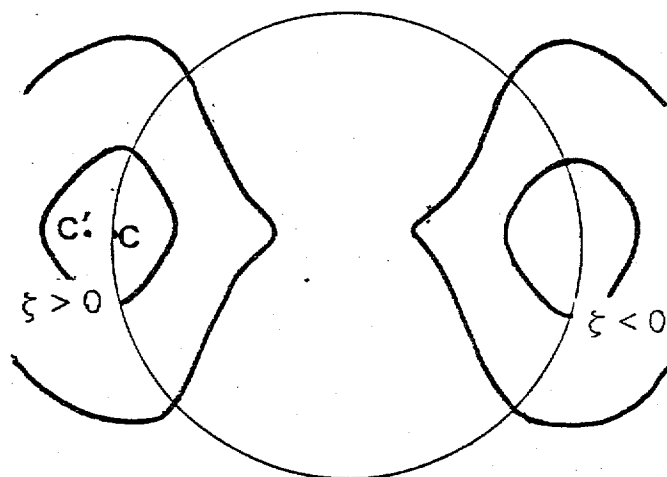


Figure 2. Schematic diagram of E-W steering gyres initially developed by symmetric vortex on a beta plane. $C'-C$ is the distance moved by the storm, in six hours under the assumptions in section 2.

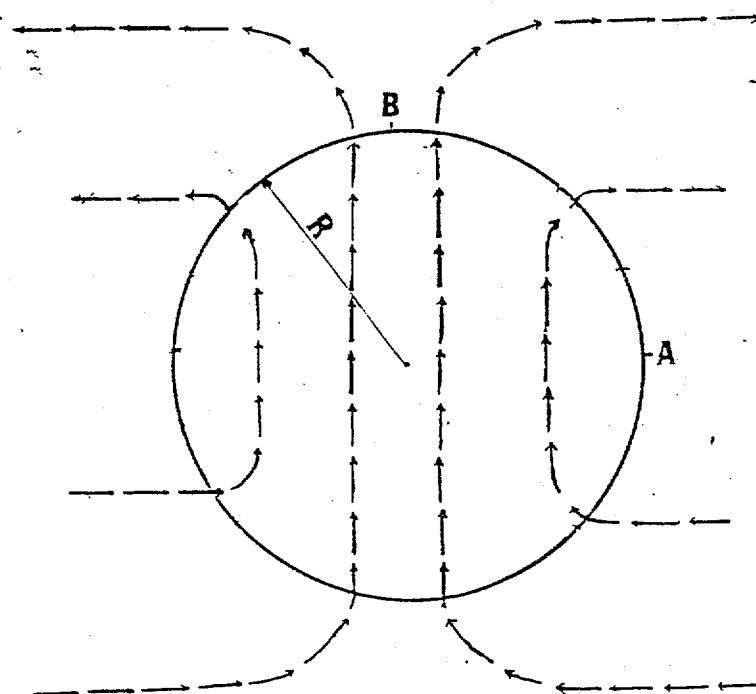


Figure 3. Schematic diagram of northward wind developed by the vorticity pattern of Figure 2.

$$\partial \xi / \partial r = -(3/4)(V_s / R^2) \quad (5)$$

We can combine equations (3) and (5) into (4) to get the speed of the westward motion. Taking a typical case with $V_s = 17$ m/s at $R = 200$ km, then the westward speed is:

$$S_w = (17)(2.15 \times 10^{-11}) / \frac{(17 \times 3)}{(2 \times 10^5)^2 \times 4} = 1.15 \text{ m/s} \quad (6)$$

Under the assumptions outlined, this westward motion is a constant, independent of time.

Now, let's examine the relative vorticity at a point 200 km west of the storm center after 6 hours (called point C in Figure 2). Assume that $\beta = 2.15 \times 10^{-11}$. Since the storm has moved 25 km during that six hours (1.15 m/s for 6 hours), at the initial time, point C was 225 km away from the center (point C' in Figure 2) and had a tangential wind of 16 m/s. If we calculate the change in relative vorticity due to β for the average wind at the point C $(16 \text{ m/s} + 17 \text{ m/s})/2$ then

$$\delta \xi = \int_0^{6 \text{ hours}} \frac{\partial \xi}{\partial t} dt = \int_0^{21600 \text{ s}} (16.5)(2.15 \times 10^{-11}) dt = 7.66 \times 10^{-6}$$

and similarly for the point to the east of the center. Since vorticity and wind are related,

$$\xi_r = \partial u / \partial y + \partial v / \partial x,$$

the wind appropriate to this vorticity pattern (as shown in Figure 3) may be found by integration:

$$\delta W_n = 2 \int_0^{200 \text{ km}} \delta \xi dr = 3.1 \text{ m/s} \quad (7)$$

where W_n denotes a wind component toward the north.

This approximates the average northward wind over the area of the storm, which would be generated by the model. The northward motion generated by the β effect is therefore three times the westward motion after only 6 hours. The northward motion will continue to increase indefinitely since it is a function of time. Thus, the "beta effect", if handled in this way, can impart a strong northward motion to the storm. Obviously, neither the real atmosphere nor the model allows this type of unlimited acceleration. This effect seems to take place during the first 12 hours of the model forecast, until

compensating forces are established. It appears that a symmetric spinup does not have the proper compensating forces. The purpose of this study is to evaluate those forces and include them in the initial conditions.

3.0 The Westward Motion

The above effect is not real, rather it is an initialization problem caused by using a symmetric vortex in the spinup. The error in the analysis becomes clear if we remember that absolute vorticity is being conserved, not earth vorticity. At the initial time, the relative vorticity is symmetric and therefore drops out of equation (1). However; within a short time the relative vorticity is no longer symmetric and must be included in equations (1) and (3). Thus the relative vorticity at any point can only increase until the change in relative vorticity in the N-S direction is equal and opposite the change in earth vorticity, i.e.,

$$\frac{\partial \zeta_r}{\partial n} = -\beta$$

This means that the relative vorticity reaches a steady state when:

$$\zeta_r = -r\beta \cos\theta$$

Note that $r\cos\theta$ is the northward displacement, from the center, of a point at radius r .

At this time, a westward wind may be calculated:

$$\delta Ww = 2 \int_0^{200\text{km}} \delta \zeta dr = 1.72\text{m/s} \quad (8)$$

where Ww denotes a wind component toward the west.

This is the correct formulation (see Figure 4) and should be included in the spinup. The time necessary to reach this steady state can be approximated by the time it takes a parcel of air, in the storm, to traverse the distance from the northernmost point to the southernmost point. Again taking the radius to be 200km, and the wind speed to be 17m/s, this time is 10.3 hours.

Figure 1 shows the results of both effects in the 0-12h time period. The northward bias occurs during the first 12 hours, as the (transient) steering gyres are developed east and west (Figure 3) of the storm. The model turns west as the (steady state) north-south gyres (Figure 4) are developed.

The above analysis neatly explains the behavior characteristics of the NMC numerical models. The initial northward motion, followed by a turning to the west is exactly as predicted. In the model, the net result is even more pronounced than indicated here. The change in vorticity for point C will occur at the rate,

$$\frac{\partial \xi}{\partial t} = V s \beta$$

and hence the maximum value will be obtained rather quickly. As the maximum is reached, the rate of change goes to zero and the westward motion of equation (6) is reduced, the westward motion shown in Figure (4) and equation (8) has not developed yet, leaving only the northward motion of equation (7). The model is handling the beta effect properly, it is simply the unbalanced initial vortex that generates the northward motion until a balance is obtained.

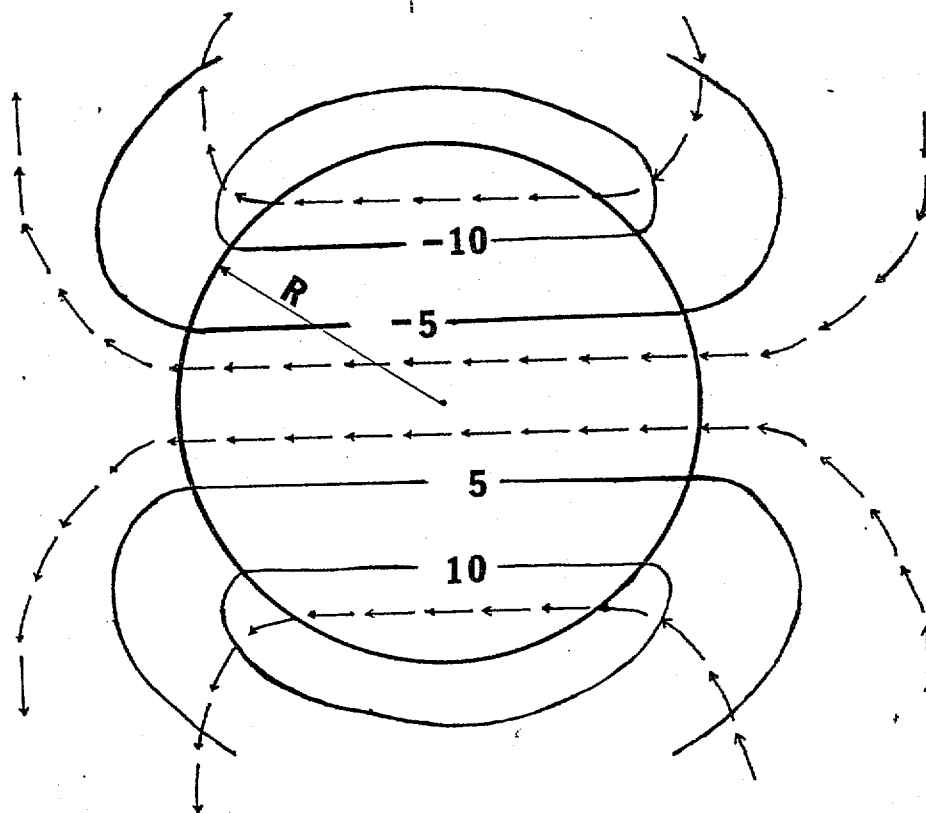


Figure 4. Schematic diagram of N-S steering gyres developed by steady state symmetric vortex on a beta plane, and the induced westward wind.

4.0 Changes and Results

The question now arises as to how to rectify this situation. The answer, of course, is to add the vorticity pattern shown in Figure 4, with a west wind over the area of the storm to the symmetric vorticity pattern currently used. This is the pattern that would be generated if a three dimensional spinup were used instead of a symmetric (2-dimensional) one. In this case, the size of the storm is the radius of the spinup, which, for the QLM, is the radius of the outermost closed isobar. If we describe the storm in polar coordinates, with r denoting the radius from the center, and θ denoting the angle, counterclockwise from north, letting R denote the maximum radius of influence of the spinup, then the asymmetric relative vorticity at point (r, θ) should be:

$$\begin{aligned}\zeta_r &= -r\beta \cos \theta && \text{for } 0 < r \leq R \\ \zeta_r &= -r\beta \cos \theta (2.0 - r/R) && \text{for } R < r \leq 2R \\ \zeta_r &= 0 && \text{elsewhere}\end{aligned} \tag{9}$$

Since relative vorticity cannot be added directly to the initial fields, it is necessary to generate wind fields that yield an equivalent vorticity change. If the large-scale fields contain no circulation, equation (9) should be applied over the entire area influenced by the spinup. The specification of the relative vorticity outside the area of influence only provides a smooth transition back to the background flow. The winds that correspond to this vorticity pattern can be calculated by relaxation, setting the winds at the boundaries to zero, in exactly the same way that heights are calculated from winds. This wind field is then added to the symmetric wind field to get the initial vortex. This change should be applied to all layers showing cyclonic circulation. Air parcels on circular trajectories will then experience relatively little change in absolute vorticity and erroneous motions will not be generated. The determination of R is straight-forward. It is the radius of the spinup used. For the QLM, this is the radius of the outermost closed isobar.

Several tests have been run using this technique at NMC. As an example, Figure 5 shows the forecast track, for the same case as Figure 1 but using the asymmetric approach. The improvement in the first 12 hours is dramatic. The model behaved exactly as the theory predicted. The storm starts out, and continues moving west along with the large-scale flow. The 12h forecast error is reduced from 142 nautical miles to 30 nautical miles, the 72h forecast is reduced from 257 nautical miles to 114 nautical miles.

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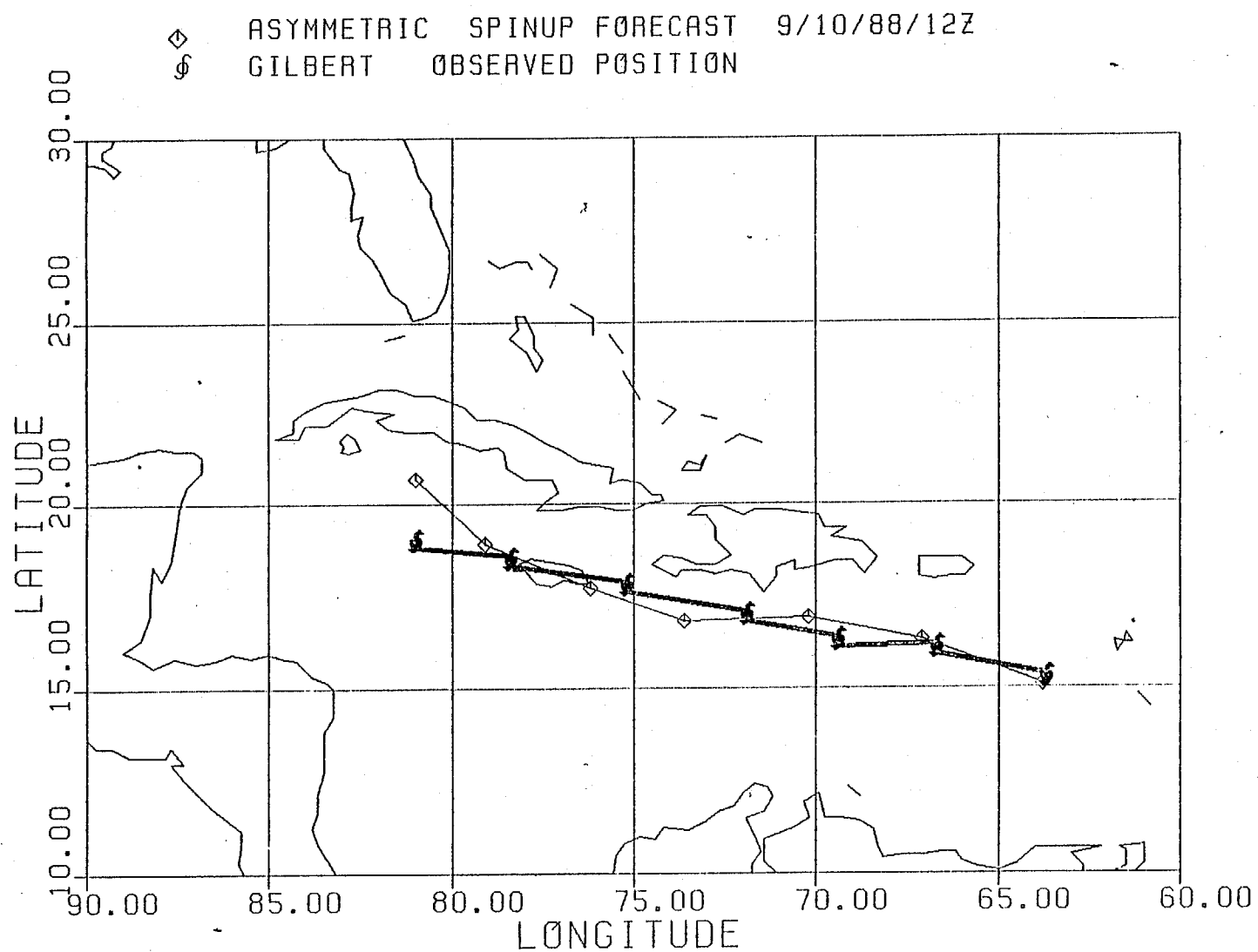


Figure 5. Gilbert 88/9/10/12z. QLM forecast from same case as Figure 1, but using asymmetric spinup.

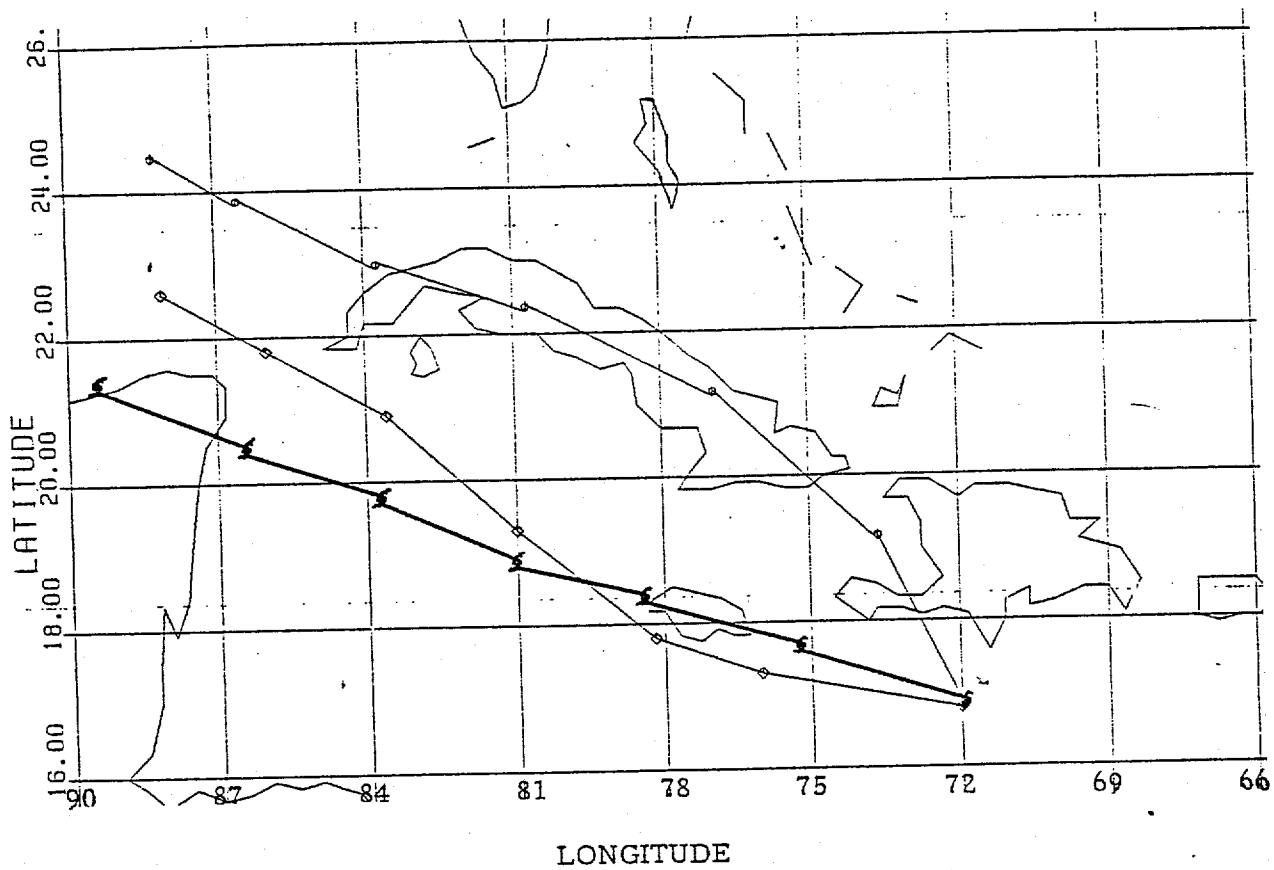


Figure 6. Gilbert 88/9/12/00z.

- Observed track
- ◊ QLM forecast with symmetric spinup
- ◊ QLM forecast with asymmetric spinup

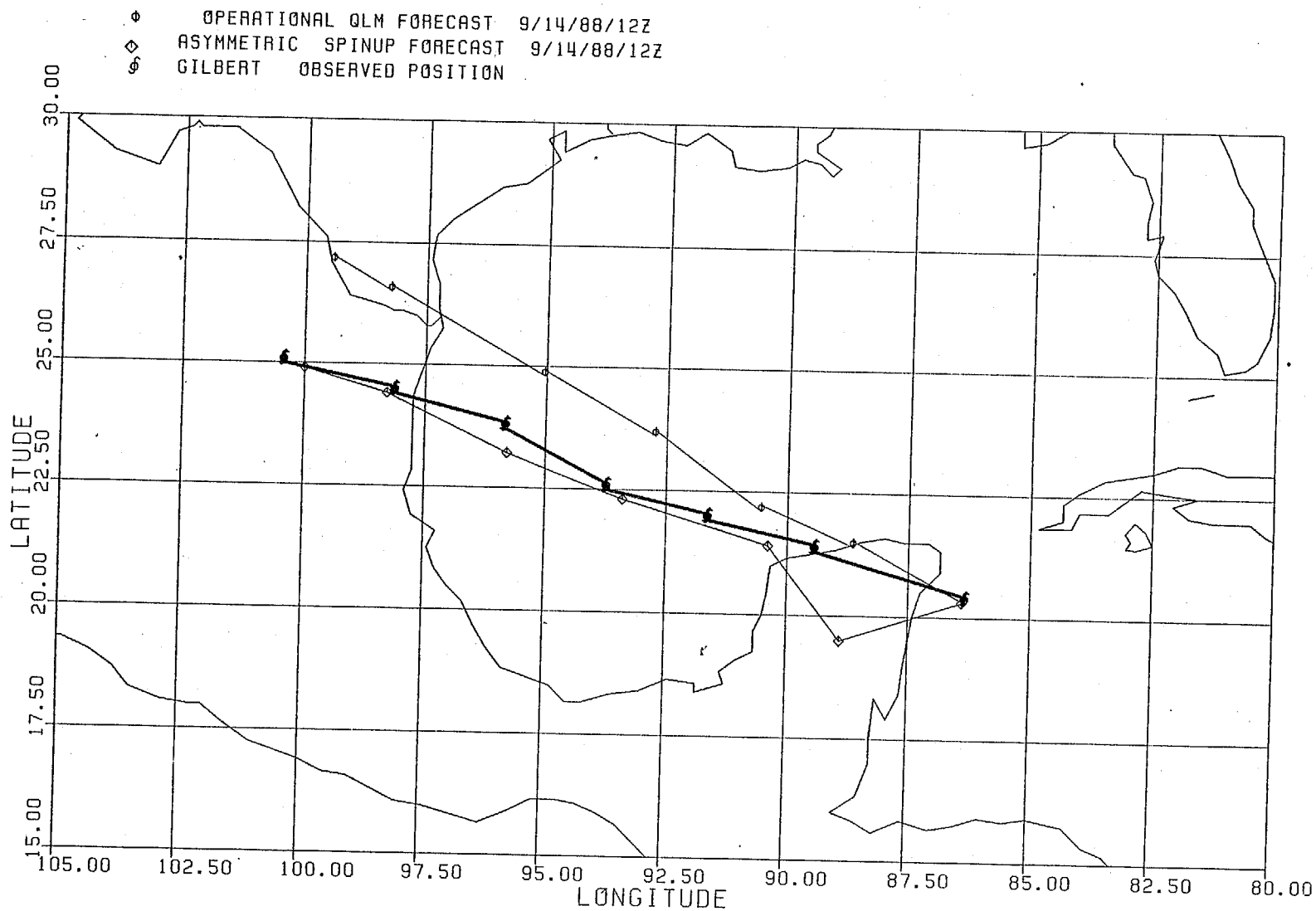
It was not expected that the longer forecasts would be affected, but they also have been improved substantially. The problems in the model or the large-scale flow field that eventually turned the forecast northward are still evident. They are not as serious since the initial error has been reduced, however. A second case, from September 12, 00z is shown in Figure 6 and the forecasts with both the old and new spinups are compared. This case also shows a substantial improvement in the forecast due to the change in spinup. The forecasts are stable, requiring no additional smoothing or adjustments, merely the addition of the wind field (Figure 4) for all lower layers (up to 300mb).

The case from 9/14/12z presented somewhat different problems. Here, the QLM already had a good forecast and the addition of the steering gyres drove the storm to the south (Figure 7). Even for this case, the longer forecasts are considerably better than the operational case. The reason for the initial southward motion appears to be that the storm, by this time, was big enough to be represented in the large scale flow. Thus the gyres of Figure 4 already existed and the "correction" made them too large. This is an unusual situation, since most tropical storms are too small to be analyzed by the global forecast system. The size of the spinup used in the 9/14/12z case was approximately 750 km. Additional, although indirect, evidence supporting this explanation comes from running the QLM without any spinup initialization. For the early cases, when the storm was too small to be included in the analysis, running without a spinup produces a more southerly track. For the later cases, the change is negligible.

The forecast shown here for 9/14/12z changed only the innermost 200km. Even that appears to have been too large. A simple test to determine the amount of vorticity already present in the background flow would be appropriate, however. Other possibilities also exist to adjust for the occasional case where a southward motion occurs, but in general, even without such safeguards, the technique will improve the forecast more often than it harms.

Although a few minor refinements to the technique may need to be worked out, the simple matter of adding an asymmetric vorticity pattern has succeeded in reducing the QLM forecast errors by one-half. This is a gigantic change, far exceeding what was expected. It is always difficult to draw conclusions from such a few cases, and modifications to the technique may have to be made for northward moving storms. However, the technique holds promise for significant improvements to other numerical models that use similar methods. A more thorough understanding of this effect may also lead to better understanding of hurricane dynamics in general.

Figure 7. Gilbert 88/9/14/12z.



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